Special Issue on
Parameterized and Approximation
Algorithms in Graph Drawing
Guest Editors’ Foreword

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1 Parameterized and Approximation Algorithms in Graph Drawing

Graph Drawing is a fundamental area of study in computer science that focuses on visually representing graphs to facilitate their understanding and analysis. The field encompasses various subdomains, including parameterized algorithms and approximation algorithms, which play crucial roles in addressing the efficiency and optimality of graph drawings. The interplay between parameterized algorithms, approximation algorithms, and graph drawing lies in their shared objective of efficiently generating visually appealing and informative graph drawings. Parameterized algorithms leverage graph structure to optimize the drawing process, while approximation algorithms provide computationally feasible solutions that approximate the optimal drawing quality. These techniques offer valuable tools for tackling the complexities of graph drawing problems, facilitating effective visual analysis and communication in various domains, including network visualization, social networks, biological networks, large-scale integrated circuit layouts, and information visualization.

Graph Drawing. Graphs are not only a common tool for modeling and solving problems in computer science, but are also often used for visualizing data. Concrete drawings of graphs are understood also by non-experts; the representation of a link or a connection is intuitive. Methods for graph drawing can be used both to represent abstract relational data and to visualize real networks such as metro networks. The goal is to create drawings that convey the structural properties and relationships within the graph, allowing users to intuitively grasp its characteristics. Graph drawing techniques encompass a wide range of approaches, including force-directed algorithms, hierarchical layouts, orthogonal layouts, and circular drawings, among others. The choice of a drawing method depends on the specific requirements of the application domain and on the desired visual representation.

Parameterized Algorithms. Parameterized algorithms are designed to efficiently solve computational problems by incorporating a parameter that quantifies the complexity or structure of the problem instance. Parameterized algorithms aim to provide solutions that are not only correct, but also efficiently computable for instances where the identified parameter is small. In particular, in graph drawing, parameterized algorithms can exploit structural properties of the graph to optimize the layout quality while maintaining computational efficiency. By considering parameters such as the treewidth or vertex degrees, these algorithms can achieve drawings that exhibit desirable properties, such as few edge crossings or small area requirements.

Approximation Algorithms. Approximation algorithms, on the other hand, focus on finding solutions that are close to optimal when it is computationally challenging to determine the exact optimal solution. In graph drawing, where the optimal drawing often depends on specific criteria, approximation algorithms provide feasible solutions that come within a certain factor of the optimal solution. By sacrificing optimality for efficiency, approximation algorithms enable the creation of visually appealing and readable drawings in a reasonable amount of time. These algorithms are particularly useful when exact solutions are computationally infeasible or when the problem is known to be NP-hard. Approximation algorithms in graph drawing offer a complementary approach by providing near-optimal solutions within acceptable computational bounds. By balancing trade-offs between visual quality and runtime, these algorithms enable the creation of aesthetically pleasing drawings that capture the essence of the graph’s structure.
2 Contributions

This special issue contains four papers that went through a thorough refereeing and revision process. The papers of this special issue cover a broad range of topics of interest to parameterized and approximation algorithms in graph drawing and reflect the state of the art on this topic. The papers appear here in alphabetical order of the last names of the first authors. We briefly introduce all papers.

• Arrighi, Fernau, de Oliveira Oliveira, and Wolf study order configurations under width constraints. The article discusses the field of reconfiguration, which involves studying relationships between solutions of a problem instance. It studies three fundamental questions in reconfiguration: whether any two solutions can be transformed into each other, whether they can be transformed in a polynomial number of steps, and whether a polynomial-time algorithm can find a transformation sequence between two given solutions. The paper focuses on reconfiguration problems in the context of linear arrangements of graph vertices, providing results on the reconfiguration of linear orders with a specific width constraint. Furthermore, it establishes connections between reconfiguration problems and two seemingly unrelated computational problems: reachability in two-letter string rewriting systems and graph isomorphism. The connections are established through the concepts of slices, unit decompositions, and rewriting rules. The paper presents results regarding the reconfiguration of linear arrangements and its implications for the relationship between string rewriting and graph isomorphism problems.

• Arroyo and Felsner study approximation algorithms for the bundled crossing number. The study of bundled crossings in Graph Drawing aims to improve the readability of graph representations by bundling crossings into grid patterns instead of minimizing individual crossings. The bundled crossing number is defined as the minimum number of bundles required to represent a graph drawing, where a bundle is a subgraph isomorphic to an $n \times m$-grid graph consisting exclusively of crossings. The authors present a polynomial-time algorithm for computing an 8-approximation of the bundled crossing number in a good graph drawing with an additive term depending on the number of special face types (toothed holes). The approach also achieves an 8-approximation in circular drawings and for families of pseudosegments, and a $\frac{9}{2}$-approximation when the intersection graph of pseudosegments is bipartite and has no toothed hole.

• Chaplick, Fleszar, Lipp, Ravsky, Verbitsky, and Wolff study the computational complexity of affine cover numbers, which measure the minimum number of geometric objects needed to cover a graph in a specific dimension. Specifically, the authors consider the line cover numbers $\rho_1^2(G)$ and $\rho_3^3(G)$, which represent the minimum number of lines required to cover a graph in 2D and 3D, respectively. They prove that computing these line cover numbers is $\exists \mathbb{R}$-hard and establish their fixed-parameter tractability. The realizability of $\rho_2^2$-optimal drawings is explored, showing that some optimal drawings require irrational coordinates. The paper also investigates the computational complexity of the plane cover number $\rho_3^2(G)$, demonstrating its NP-hardness.

• Chimani and Hliněný study the problem of inserting multiple edges into a planar graph. The paper addresses the computation of the crossing number in graph drawing, a challenging optimization problem. It discusses parameterized algorithms and approximation algorithms as approaches to tackle the problem. The focus is on the Multiple Edge Insertion (MEI)
problem: given a plane graph $G$ and a set of edges $F$, draw $G + F$ such that $G$ is planar and the total number of crossings is minimized. The paper proposes exact, linear-time algorithms for finding a planar drawing with minimum crossings when $F$ has constant size, both for the general MEI problem and the rigid MEI problem, where the initial drawing of $G$ remains fixed.

We thank the authors for preparing, submitting and revising their papers, the referees for their careful reviews, and the Editorial Board of the Journal of Graph Algorithms and Applications for making this special issue possible.